NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If $\Delta = \begin{vmatrix} 3 & 4 & 5 & x \\ 4 & 5 & 6 & y \\ 5 & 6 & 7 & 8 \\ x & y & z & 0 \end{vmatrix}$, then find the value of Δ.

b) For what value of λ, the system of equations

$$X + y + z = 6$$
$$X+2y+3z = 10$$
$$X +2y + \lambda z = 12$$

is in consistent.

c) If A = $\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3|$ =125 then find the value of α.

d) If a = i + j +k, b = 4i + 3 j + 4 k ad c = i + αj +βk are linearly dependent vectors and |c| = $\sqrt{3}$ find the values of α and β.

e) Find the Eigen values of the matrix:

$$A = \begin{vmatrix} 6 & 2 & 2 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

f) Find the rank of the matrix

$$\begin{vmatrix} 3 & 1 & -2 \\ 2 & 0 & -1 \\ 1 & 4 & 1 \end{vmatrix}$$

g) If a = 3i - j +2k, b = 2i + j – k, c = i - 2j + 2k, find (aX b)Xc and a X (b X c). Are both same?

(7x4)

2.

a) For how many values of x in the closed interval [-4,-1], the matrix $\begin{vmatrix} 3 & -1+x & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{vmatrix}$ is singular.

b) If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find the value of I + A + A^2 + A^3 +........∞.

c) Let a = 2i +3j – k and b = I – 2j +3k.Then find the value of λ for which the vector c = λI +j + (2λ-1)k is parallel to the plane containing a and b.

(6+6+6)

3.

a) An unbiased die is thrown at random. What is the expectation of the number on it?

b) In a city, 5 accidents take place in a span of 25 days. Assuming that the number of accidents follows the poisson distribution what is the probability that there will be 3 or more accidents in a day? (Given e^{-2} =0.8187)

c) Apply Maclaurin's expansion for expanding log(1+x) and log (1-x) in ascending powers of x and hence deduce the expansion of log ((1+x)/(1-x)).

(6+6+6)

4.

a) Find the covariance between x and y for the following observation (x,y).

X:	3	11	21	11	18	13	16	17
Y:	5	16	13	17	18	11	12	9

b) Find the limit of the following:

$$\lim_{n\to\infty} \frac{7n^3 - 8n^2 + 10n - 7}{8n^3 - 9n^2 + 5} -$$

c) Find the extreme values of the function $x^3 e^{-x}$.

(6+6+6)

5.

a) Integrate $\sqrt{(1+\sin 2x)}$ with respect to x.

b) Evaluate $I = \int_0^{\pi/2} \log(\sin x)\, dx$.

c) State the values of a and b if the equation
$a x^2 + 2 b x y - 2 y^2 + 8 x + 12 y + 6 = 0$
represents a circle. Substituting the values of these a and b in the equation, find the center and radius of the circle

(6+6+6)

6.

a) Integrate $\frac{x^3}{(x^2+1)^3}$ with respect to x.

b) Consider the function defined as follows:

$$f(x) = \frac{x^2 - 4}{x - 2} \quad for\ x < 2$$

$$f(x) = 4 \quad for\ x = 2$$

$$f(x) = 2, \ for\, x > 2$$

Discuss the continuity at x = 2.

c) If $C_0, C_1, C_2 \ldots C_n$ denote the coefficients of the expansion of $(1+x)^n$, prove that

i) $C_1 + 2C_2 + 3C_3 + \ldots + nC_n = n\,2^{n-1}$

ii) $C_0 + 2C_1 + 3C_2 + \ldots + (n+1)C_n = (n+2)\,2^{n-1}$

iii) $C_0 + 3C_1 + 5C_2 + \ldots + (2n+1)C_n = (n+1)2^n$

(6+6+6)

7.

a) Find the equation of the straight line passing through the intersection of 4x – 3 y – 1 = 0 and 2x – 5 y + 3 = 0 and

i) Parallel to 4x +5y = 6

ii) Perpendicular to 2x +3y = 12

b) Explain following;

i) Polar coordinates

ii) Fundamental theorem of calculus

iii) L. Hospital's rule

iv) Applications of Eigen values

v) Conics and their classification

(9+9)