C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Six men in a company of 15 are doctors. If 3 men are picked out of 15 at random, find the probability that at least one of them is a doctor.

b) Obtain the dual of the following linear programming problem:

Maximize

$$z = 3x_1 + 2x_2,$$

subject to

$$x_1 + 3x_2 \geq 1,$$
$$x_1 + x_2 \leq 7,$$
$$x_1 + 2x_2 \leq 10,$$
$$2x_2 \leq 3, \quad x_1, \; x_2, \; \geq 0.$$

c) The transition probability matrix of a Markov chain having three states 1, 2, 3 is

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.1 & 0.5 & 0.4 \\ 2 & 0.6 & 0.2 & 0.2 \\ 3 & 0.3 & 0.4 & 0.3 \end{array}$$

If the initial state distribution is $P^{(0)} = (0.7 \quad 0.2 \quad 0.1)$, find $P^{(1)}$ and $P\{X_2 = 3\}$.

d) Let X be a random variable with following probability distribution

$$\begin{array}{lcccccc} X & : & -1 & 0 & 1 & 2 \\ P(X) & : & \frac{1}{3} & \frac{1}{4} & \frac{1}{6} & \frac{1}{4} \cdot \end{array}$$

Find expected values of X and X^2.

e) Let X be a random variable taking on a finite number of values. Find the inequality relationship of H(X) and H(Y) when Y = cos X.

f) Draw the transition diagram of the process whose one step transition matrix with three states is as follows:

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 0.5 & 0.5 \\ 2 & 0.4 & 0.6 & 0 \\ 3 & 0.2 & 0.2 & 0.6 \end{array} .$$

g) Find the probability distribution of number of heads when 3 coins are tossed simultaneously.

(7×4)

2.

a) Expand $f(x) = x \sin x$ in the interval, $0 < x < 2\pi$ as a Fourier series.

b) Use simplex method to solve:

Maximize $z = x_1 + 3x_2$,

subject to

$-x_1 + 2x_2 \leq 2$,

$x_1 - 2x_2 \leq 2$,

$x_1, x_2 \geq 0$.

(9+9)

3.

a) Find the inverse Laplace Transform of $(2s + 1)/\{(s + 2)^2 (s - 1)^2\}$.

b) Let p(x, y) be given by

Y→ X ↓	0	1
0	1/3	1/3
1	0	1/3

Find (i) H(X), H(Y) (ii) H(X/Y), H(Y/X) (iii) H(X, Y) and (iv) I(X; Y).

(9+9)

4.

a) In a railway marshalling yard, goods train arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes, calculate: (i) the mean queue size. (ii) the probability that the queue size exceedes 10. (iii) If the input of trains increases to an average 33 per day, what will be the change in (i) and (ii).

b) A salesman visits three cities A, B and C regarding sale of his products. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B, however, if he sells in B or C, then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in steady state?

(9+9)

5.

a) Use the KKT conditions to derive an optimal solution for the following convex programming problem:

Maximize

$$f(x_1, x_2) = 12x_1 - x_1^2 + 50x_2 - x_2^2,$$

subject to

$$x_1 \leq 10,\ x_2 \leq 15 \text{ and } x_i \geq 0 \text{ for } i = 1, 2.$$

b) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed according to a Poisson process with average rate of 15 letters per hour, find (i) the probability of all the typists to be busy, (ii) the average number of letters waiting to be typed and (iii) the average time a letter has to spend for waiting and for being typed.

(9+9)

6.

a) The joint PDF of (X, Y) is given by

$$f(x,y)=\begin{cases} kxy, & 1\le x\le 3,\ 2\le y\le 4, \\ 0, & elsewhere. \end{cases}$$

Find (i) the value of *k*, (ii) the conditional densities of Y given X and X given Y.

b) Use dynamic programming to solve the following problem:

Minimize $z = x_1^2 + x_2^2 + x_3^2$

subject to the constraints

$$x_1 + x_2 + x_3 \ge 15,$$
$$x_1,\ x_2,\ x_3 \ge 0.$$

(9+9)

7.

a) If $\{X(t)\}$ is a Poisson Process, then prove that the correlation coefficient between $X(t)$ and $X(t+s)$ is $\sqrt{\frac{t}{t+s}}$.

b) If *X*(*t*) = *Asin*(*Bt* + θ), where A and B are constant and θ is a uniformly distributed random variable in $(0, 2\pi)$, calculate the mean and autocorrelation function of the process $\{X(t)\}$.

(9+9)