B0-R4 : BASIC MATHEMATICS

NOTE :
1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time : 3 Hours   **Total Marks : 100**

1. (a) Show that $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is an orthogonal matrix.

   (b) Evaluate $\lim\limits_{x \to 0} \dfrac{xe^x - \log(1+x)}{x^2}$.

   (c) Find the angle between two vectors $A = 4\vec{i} - 2\vec{j} - \vec{k}$ and $B = 4\vec{i} - 2\vec{j} + 4\vec{k}$.

   (d) Express the complex number $(2+3i)(3-4i)$ in the form $a+ib$.

   (e) Solve the differential equation $(x+1)\dfrac{dy}{dx} = x(y^2+1)$.

   (f) Test the convergence of the series $\dfrac{1}{2} + \dfrac{2!}{8} + \dfrac{3!}{32} + \ldots$

   (g) Evaluate $\int\limits_{0}^{\frac{\pi}{4}} \sqrt{1 + \cos 4x}\, dx$.   **(7x4)**

2. (a) Find the Eigen value and Eigen vectors of the matrix $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$.

   (b) Find the all five value of the expression $(1+i)^{\frac{1}{5}}$.   **(10+8)**

3. (a) Find the area of the region bounded by the curves $y = x^4$ and $y = 8x$ in the positive quadrant.

   (b) Find the Taylor series expansion for $\log_e \cos x$ about the point $\dfrac{\pi}{3}$.   **(9+9)**

4. (a) Find the value of c for the function $(x-4)^2 + 1$ on the interval $[3, 6]$ that satisfies the mean value theorem.

(b) Find the equation of the hyperbola with the vertices $(0, \pm 6)$ and $e = \dfrac{5}{3}$ and also find its foci.

(c) Test the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{n+2}{(n+1)\sqrt{n}}$.   **(6+6+6)**

5. (a) Solve, with the help of matrices the simultaneous equations
$x + 2y + 3z = 4,\ x + 4y + 9z = 6,\ x + y + z = 3$.

(b) Evaluate $\displaystyle\int \dfrac{xe^x}{(x+1)^2}\,dx$.

(c) Find the values of constants a and b such that

$$\lim_{x \to 0} \frac{axe^x - b\log(1+x) + x}{x^3} = \frac{-1}{18}$$
       (6+6+6)

6. (a) Using the properties of determines, solve for x $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$.

(b) Solve the following differential equation :
$(1 + y^2)dx = (\tan^{-1} y - x)dy$

(c) Find the centre, the length of the axes and the eccentricity of the ellipse
$2x^2 + 3y^2 - 4x - 12y + 13 = 0$.   **(6+6+6)**

7. (a) Test the convergence of the series $1 - \dfrac{1}{2\sqrt{2}} + \dfrac{1}{3\sqrt{3}} - \dfrac{1}{4\sqrt{4}} + \ldots$

(b) Find the volume of the solid generated by revolving the region enclosed by
$x = \dfrac{2}{y},\ y = 2, y = 6$ about the y-axis.

(c) Find inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by elementary transformations.   **(6+6+6)**

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