

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

- a) For matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, verify that $A^2 - 5A + I = O$, where I is a unit matrix of order 2, hence find A^{-1} .
- b) Evaluate $\lim_{n \rightarrow \infty} \left[\frac{1^2}{n^3} + \frac{2^2}{n^3} + \frac{3^2}{n^3} + \dots + \frac{n^2}{n^3} \right]$.
- c) Evaluate $\int_0^{\sqrt{2}} (\sqrt{2-x^2}) dx$.
- d) Solve the differential equation $\frac{dy}{dx} = \frac{1+\cos x}{1-\cos x}$.
- e) Express $\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}$ in the form of $a + ib$.
- f) Find the area of a parallelogram whose diagonals are represented by the vectors $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.
- g) Find the asymptotes of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$.

(7x4)

2.

- a) Reduce the following matrix into its normal form and hence find its rank.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -1 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$

- b) Test the convergence of the series

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots \dots \dots \infty.$$

(9+9)

3.

- a) Find the Eigenvalues and Eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

- b) Evaluate the integral $\int \frac{dx}{5+4 \cos x}$.

(10+8)

4.

- a) Find the area of the region $\{(x, y) : x^2 + y^2 \leq 2ax, y^2 \geq ax, x \geq 0, y \geq 0\}$.
- b) Solve the differential equation: $x^2(y-1)dx + y^2(x-1)dy = 0$.

(10+8)

5.

- a) Find the local maxima and local minima for the function: $f(x) = x^4 - 62x^2 + 120x + 9$.
- b) If $f(x) = (\cos x)^{\log x} + (\log x)^x$, find $f'(x)$.

(9+9)

6.

- a) Find the equation of the plane through the points (4, 5, 1), (3, 9, 4) and (-4, 4, 4).
b) Find a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ where $\vec{a} = 3i + 2j + 5k$ and $\vec{b} = i - 3j + k$. Find the angle between $\vec{a}$ and $\vec{b}$.
(9+9)

7.

- a) Find the equation of the ellipse whose minor axis is equal to the distance between the foci and latus rectum is 10.
b) Expand $\tan\left(x + \frac{\pi}{4}\right)$ as far as the term x^4 and evaluate $\tan 46.5^\circ$ to four significant digits.
(9+9)
