

**B41-R4: COMPUTER BASED NUMERICAL AND STATISTICAL TECHNIQUES****NOTE :**

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.
3. Probability Tables are allowed.
4. Only Non-Programmable and Non-Storage type Scientific Calculator is allowed.

**Total Time : 3 Hours****Total Marks : 100**

1. (a) Three approximate values of the number  $\frac{1}{3}$  are given as 0.30, 0.33 and 0.34. Which of these values is the best approximation ?
- (b) Four letters of the word "THURSDAY" are arranged in all possible ways. Find the probability that the word formed "HURT".
- (c) Calculate  $\text{Var}(X)$  if  $X$  represents the outcome when a fair die is rolled.
- (d) Use the Gauss elimination method to solve the following equations :  
 $x + 4y - z = -5$ ,  $x + y - 6z = -12$ ,  $3x - y - z = 4$
- (e) The first three moments of a distribution about the value "2" of the variable are 1, 16 and  $-40$ . Show that the mean is 3, variance is 15 and  $\mu_3 = -86$ .
- (f) From the following table, find the value of  $\frac{dy}{dx}$  at  $x = 2.03$ .

|     |        |        |        |        |        |
|-----|--------|--------|--------|--------|--------|
| $x$ | 1.96   | 1.98   | 2.00   | 2.02   | 2.04   |
| $y$ | 0.7825 | 0.7739 | 0.7651 | 0.7563 | 0.7473 |

- (g) In a Binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter  $P$  of the distribution.

**(4x7)**

2. (a) 200 digits were chosen at random from a set of tables. The frequency of the digits are shown below :

|        |    |    |    |    |    |    |    |    |    |    |
|--------|----|----|----|----|----|----|----|----|----|----|
| Digits | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| Freq.  | 18 | 19 | 23 | 21 | 16 | 25 | 22 | 20 | 21 | 15 |

Use the Chi-square test to access the correctness of the hypothesis that the digits were distributed in equal number in the tables from which these were chosen.

- (b) If  $P$  is the pull required to fit a load  $W$  by means of pulley block, find a linear law of the form  $P = mW + c$  connecting  $P$  and  $W$  using the following data :

|     |    |    |     |     |
|-----|----|----|-----|-----|
| $P$ | 12 | 15 | 21  | 25  |
| $W$ | 50 | 70 | 100 | 120 |

Also compute  $P$  when  $W = 150$  kg.

(10+8)

3. (a) A total of 48 percent of the women and 37 percent of the men who took a certain "quit smoking" class remained non-smokers for at least one year after completing the class. These people then attended a success party at the end of the year. If 62 percent of the original class was male.

- (i) what percentage of those attending the party were women ?  
(ii) what percentage of the original class attended the party ?

- (b) Solve the following system of linear equations :

$$3x - 0.1y - 0.2z = 7.85,$$

$$0.1x + 7y - 0.3z = -19.3,$$

$$0.3x - 0.2y + 10z = 71.4$$

Using Gauss-Seidel method.

(9+9)

4. (a) The daily consumption of milk in a city, in excess 20000 liters is approximately distributed as a Gamma variate with parameter  $\lambda = \frac{1}{10000}$  and  $r = 2$ . The city has a daily stock of 30000 liters. What is the probability that the stock is insufficient on a particular day ?

- (b) Of three persons the chances that politician, a businessman, or an academician would be appointed the Vice Chancellor (VC) of a university are 0.5, 0.3, 0.2 respectively. Probabilities that research is promoted by these persons if they are appointed as VC are 0.3, 0.7, 0.8 respectively.

- (i) Determine the probability that research is promoted.  
(ii) If research is promoted, what is the probability that VC is an academician.

- (c) Find the real root of  $x \log_{10} x - 1.2 = 0$  correct up to three decimal places.

(6+6+6)

5. (a) Evaluate  $\int_0^{\frac{\pi}{2}} e^{\sin x} dx$  using Simpson's  $\frac{3}{8}$  rule with  $n = 6$ .

- (b) Assume that the mean height of Indian soldiers is 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 1000 would you expect to be over 6 feet tall ?

- (c) Use Chebyshev's inequality to find how many times a fair coin must be tossed in order that probability that the ratio of the number of heads to the number of tosses will be between 0.45 and 0.55 will be at least 0.95.

(6+6+6)

6. (a) For a bivariate data, the mean value of  $x$  is 20 and mean value of  $y$  is 45. The regression coefficient of  $y$  on  $x$  is 4 and that of  $x$  on  $y$  is  $\frac{1}{9}$ . Find :
- The coefficient of correlation.
  - The standard deviation of  $x$  if the standard deviation of  $y$  is 12.
  - Also write down the equations of regression lines.
- (b) A rocket is launched from the ground its acceleration is measured for every 5 seconds is tabulated below.

|      |    |       |      |       |       |       |      |      |      |
|------|----|-------|------|-------|-------|-------|------|------|------|
| t    | 0  | 5     | 10   | 15    | 20    | 25    | 30   | 35   | 40   |
| a(t) | 40 | 45.25 | 48.5 | 51.25 | 54.35 | 59.48 | 61.5 | 64.3 | 68.7 |

Using Trapezoidal rule and Simpson's  $\frac{1}{3}$  rule, find the velocity and position of rocket at  $t = 40$  seconds.

- (c) A player wins if he gets 5 on a single throw of a dice, he loses if he gets 2 or 4. If he wins, he gets ₹ 50, if he loses he gets ₹ 10 otherwise he has to pay ₹ 15. Find the value of the game to the player. Is it favorable ?

(6+6+6)

7. (a) The area  $A$  of a circle of diameter  $d$  is given for the following values.

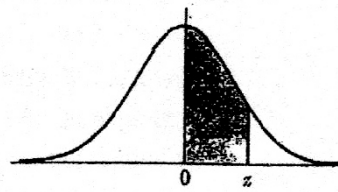
|   |      |      |      |      |      |
|---|------|------|------|------|------|
| d | 80   | 85   | 90   | 95   | 100  |
| A | 5026 | 5674 | 6362 | 7088 | 7854 |

Calculate the area of a circle of a diameter of 98 units.

- (b) Suppose a book of 585 pages contains 43 typographical errors. If these errors are randomly distributed throughout the book, what is the probability that 10 pages, selected at random, will be free from errors ?
- (c) Suppose that each child born is equally likely to be a boy or a girl. Consider a family with exactly three children.
- List the eight elements in the sample space whose outcomes are all possible genders of the three children.
  - Write each of the following events as a set and find its probability :
    - The event that exactly one child is a girl.
    - The event that at least two children are girls
    - The event that no child is a girl.

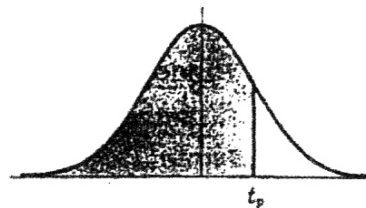
(6+6+6)

Areas  
Under the  
Standard  
Normal Curve  
from 0 to  $z$



| $z$ | 0     | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0754 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2258 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2518 | .2549 |
| 0.7 | .2580 | .2612 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2996 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 3.1 | .4990 | .4991 | .4991 | .4991 | .4992 | .4992 | .4992 | .4992 | .4993 | .4993 |
| 3.2 | .4993 | .4993 | .4994 | .4994 | .4994 | .4994 | .4994 | .4995 | .4995 | .4995 |
| 3.3 | .4995 | .4995 | .4995 | .4996 | .4996 | .4996 | .4996 | .4996 | .4996 | .4997 |
| 3.4 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4998 |
| 3.5 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 |
| 3.6 | .4998 | .4998 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 |
| 3.7 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 |
| 3.8 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 |
| 3.9 | .5000 | .5000 | .5000 | .5000 | .5000 | .5000 | .5000 | .5000 | .5000 | .5000 |

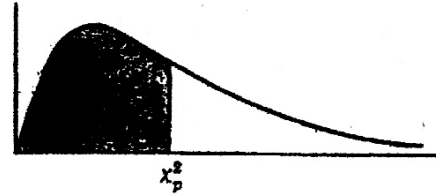
Percentile Values ( $t_p$ )  
for  
Student's  $t$  Distribution  
with  $\nu$  Degrees of Freedom  
(shaded area =  $p$ )



| $p$      | $t_{.995}$ | $t_{.99}$ | $t_{.975}$ | $t_{.95}$ | $t_{.90}$ | $t_{.80}$ | $t_{.75}$ | $t_{.70}$ | $t_{.60}$ | $t_{.55}$ |
|----------|------------|-----------|------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1        | 63.66      | 31.82     | 12.71      | 6.31      | 3.08      | 1.376     | 1.000     | .727      | .325      | .158      |
| 2        | 9.92       | 6.96      | 4.30       | 2.92      | 1.89      | 1.061     | .816      | .617      | .289      | .142      |
| 3        | 5.84       | 4.54      | 3.18       | 2.35      | 1.64      | .978      | .765      | .584      | .277      | .137      |
| 4        | 4.60       | 3.75      | 2.78       | 2.13      | 1.53      | .941      | .741      | .569      | .271      | .134      |
| 5        | 4.03       | 3.36      | 2.57       | 2.02      | 1.48      | .920      | .727      | .559      | .267      | .132      |
| 6        | 3.71       | 3.14      | 2.45       | 1.94      | 1.44      | .906      | .718      | .553      | .265      | .131      |
| 7        | 3.50       | 3.00      | 2.36       | 1.90      | 1.42      | .896      | .711      | .549      | .263      | .130      |
| 8        | 3.36       | 2.90      | 2.31       | 1.86      | 1.40      | .889      | .706      | .546      | .262      | .130      |
| 9        | 3.25       | 2.82      | 2.26       | 1.83      | 1.38      | .883      | .703      | .543      | .261      | .129      |
| 10       | 3.17       | 2.76      | 2.23       | 1.81      | 1.37      | .879      | .700      | .542      | .260      | .129      |
| 11       | 3.11       | 2.72      | 2.20       | 1.80      | 1.36      | .876      | .697      | .540      | .260      | .129      |
| 12       | 3.06       | 2.68      | 2.18       | 1.78      | 1.36      | .873      | .695      | .539      | .259      | .128      |
| 13       | 3.01       | 2.65      | 2.16       | 1.77      | 1.35      | .870      | .694      | .538      | .259      | .128      |
| 14       | 2.98       | 2.62      | 2.14       | 1.76      | 1.34      | .868      | .692      | .537      | .258      | .128      |
| 15       | 2.95       | 2.60      | 2.13       | 1.75      | 1.34      | .866      | .691      | .536      | .258      | .128      |
| 16       | 2.92       | 2.58      | 2.12       | 1.75      | 1.34      | .865      | .690      | .535      | .258      | .128      |
| 17       | 2.90       | 2.57      | 2.11       | 1.74      | 1.33      | .863      | .689      | .534      | .257      | .128      |
| 18       | 2.88       | 2.55      | 2.10       | 1.73      | 1.33      | .862      | .688      | .534      | .257      | .127      |
| 19       | 2.86       | 2.54      | 2.09       | 1.73      | 1.33      | .861      | .688      | .533      | .257      | .127      |
| 20       | 2.84       | 2.53      | 2.09       | 1.72      | 1.32      | .860      | .687      | .533      | .257      | .127      |
| 21       | 2.83       | 2.52      | 2.08       | 1.72      | 1.32      | .859      | .686      | .532      | .257      | .127      |
| 22       | 2.82       | 2.51      | 2.07       | 1.72      | 1.32      | .858      | .686      | .532      | .256      | .127      |
| 23       | 2.81       | 2.50      | 2.07       | 1.71      | 1.32      | .858      | .685      | .532      | .256      | .127      |
| 24       | 2.80       | 2.49      | 2.06       | 1.71      | 1.32      | .857      | .685      | .531      | .256      | .127      |
| 25       | 2.79       | 2.48      | 2.06       | 1.71      | 1.32      | .856      | .684      | .531      | .256      | .127      |
| 26       | 2.78       | 2.48      | 2.06       | 1.71      | 1.32      | .856      | .684      | .531      | .256      | .127      |
| 27       | 2.77       | 2.47      | 2.05       | 1.70      | 1.31      | .855      | .684      | .531      | .256      | .127      |
| 28       | 2.76       | 2.47      | 2.05       | 1.70      | 1.31      | .855      | .683      | .530      | .256      | .127      |
| 29       | 2.76       | 2.46      | 2.04       | 1.70      | 1.31      | .854      | .683      | .530      | .256      | .127      |
| 30       | 2.75       | 2.46      | 2.04       | 1.70      | 1.31      | .854      | .683      | .530      | .256      | .127      |
| 40       | 2.70       | 2.42      | 2.02       | 1.68      | 1.30      | .851      | .681      | .529      | .255      | .126      |
| 60       | 2.66       | 2.39      | 2.00       | 1.67      | 1.30      | .848      | .679      | .527      | .254      | .126      |
| 120      | 2.62       | 2.36      | 1.98       | 1.66      | 1.29      | .845      | .677      | .526      | .254      | .126      |
| $\infty$ | 2.58       | 2.33      | 1.96       | 1.645     | 1.28      | .842      | .674      | .524      | .253      | .126      |

Source: R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research* (5th edition), Table III, Oliver and Boyd Ltd., Edinburgh, by permission of the authors and publishers.

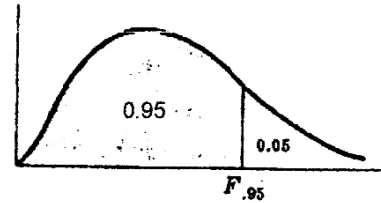
Percentile Values ( $\chi^2_p$ )  
for  
the Chi-Square Distribution  
with  $\nu$  Degrees of Freedom  
(shaded area =  $p$ )



| $\nu$ | $\chi^2_{.995}$ | $\chi^2_{.99}$ | $\chi^2_{.975}$ | $\chi^2_{.95}$ | $\chi^2_{.90}$ | $\chi^2_{.75}$ | $\chi^2_{.50}$ | $\chi^2_{.25}$ | $\chi^2_{.10}$ | $\chi^2_{.05}$ | $\chi^2_{.025}$ | $\chi^2_{.01}$ | $\chi^2_{.005}$ |
|-------|-----------------|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|----------------|-----------------|
| 1     | 7.88            | 6.63           | 5.02            | 3.84           | 2.71           | 1.32           | .455           | .102           | .0158          | .0039          | .0010           | .0002          | .0000           |
| 2     | 10.6            | 9.21           | 7.38            | 5.99           | 4.61           | 2.77           | 1.39           | .575           | .211           | .103           | .0506           | .0201          | .0100           |
| 3     | 12.8            | 11.3           | 9.35            | 7.81           | 6.25           | 4.11           | 2.37           | 1.21           | .584           | .352           | .216            | .115           | .072            |
| 4     | 14.9            | 13.3           | 11.1            | 9.49           | 7.78           | 5.39           | 3.36           | 1.92           | 1.06           | .711           | .484            | .297           | .207            |
| 5     | 16.7            | 15.1           | 12.8            | 11.1           | 9.24           | 6.63           | 4.35           | 2.67           | 1.61           | 1.15           | .831            | .554           | .412            |
| 6     | 18.5            | 16.8           | 14.4            | 12.6           | 10.6           | 7.84           | 5.35           | 3.45           | 2.20           | 1.64           | 1.24            | .872           | .676            |
| 7     | 20.3            | 18.5           | 16.0            | 14.1           | 12.0           | 9.04           | 6.35           | 4.25           | 2.83           | 2.17           | 1.69            | 1.24           | .989            |
| 8     | 22.0            | 20.1           | 17.5            | 15.5           | 13.4           | 10.2           | 7.34           | 5.07           | 3.49           | 2.73           | 2.18            | 1.65           | 1.34            |
| 9     | 23.6            | 21.7           | 19.0            | 16.9           | 14.7           | 11.4           | 8.34           | 5.90           | 4.17           | 3.33           | 2.70            | 2.09           | 1.73            |
| 10    | 25.2            | 23.2           | 20.5            | 18.3           | 16.0           | 12.5           | 9.34           | 6.74           | 4.87           | 3.94           | 3.25            | 2.56           | 2.16            |
| 11    | 26.8            | 24.7           | 21.9            | 19.7           | 17.3           | 13.7           | 10.3           | 7.58           | 5.58           | 4.57           | 3.82            | 3.05           | 2.60            |
| 12    | 28.3            | 26.2           | 23.3            | 21.0           | 18.5           | 14.8           | 11.3           | 8.44           | 6.30           | 5.23           | 4.40            | 3.57           | 3.07            |
| 13    | 29.8            | 27.7           | 24.7            | 22.4           | 19.8           | 16.0           | 12.3           | 9.30           | 7.04           | 5.89           | 5.01            | 4.11           | 3.57            |
| 14    | 31.3            | 29.1           | 26.1            | 23.7           | 21.1           | 17.1           | 13.3           | 10.2           | 7.79           | 6.57           | 5.63            | 4.66           | 4.07            |
| 15    | 32.8            | 30.6           | 27.5            | 25.0           | 22.3           | 18.2           | 14.3           | 11.0           | 8.55           | 7.26           | 6.26            | 5.23           | 4.60            |
| 16    | 34.3            | 32.0           | 28.8            | 26.3           | 23.5           | 19.4           | 15.3           | 11.9           | 9.31           | 7.96           | 6.91            | 5.81           | 5.14            |
| 17    | 35.7            | 33.4           | 30.2            | 27.6           | 24.8           | 20.5           | 16.3           | 12.8           | 10.1           | 8.67           | 7.56            | 6.41           | 5.70            |
| 18    | 37.2            | 34.8           | 31.5            | 28.9           | 26.0           | 21.6           | 17.3           | 13.7           | 10.9           | 9.39           | 8.23            | 7.01           | 6.26            |
| 19    | 38.6            | 36.2           | 32.9            | 30.1           | 27.2           | 22.7           | 18.3           | 14.6           | 11.7           | 10.1           | 8.91            | 7.63           | 6.84            |
| 20    | 40.0            | 37.6           | 34.2            | 31.4           | 28.4           | 23.8           | 19.3           | 15.5           | 12.4           | 10.9           | 9.59            | 8.26           | 7.43            |
| 21    | 41.4            | 38.9           | 35.5            | 32.7           | 29.6           | 24.9           | 20.3           | 16.3           | 13.2           | 11.6           | 10.3            | 8.90           | 8.03            |
| 22    | 42.8            | 40.3           | 36.8            | 33.9           | 30.8           | 26.0           | 21.3           | 17.2           | 14.0           | 12.3           | 11.0            | 9.54           | 8.64            |
| 23    | 44.2            | 41.6           | 38.1            | 35.2           | 32.0           | 27.1           | 22.3           | 18.1           | 14.8           | 13.1           | 11.7            | 10.2           | 9.26            |
| 24    | 45.6            | 43.0           | 39.4            | 36.4           | 33.2           | 28.2           | 23.3           | 19.0           | 15.7           | 13.8           | 12.4            | 10.9           | 9.89            |
| 25    | 46.9            | 44.3           | 40.6            | 37.7           | 34.4           | 29.3           | 24.3           | 19.9           | 16.5           | 14.6           | 13.1            | 11.5           | 10.5            |
| 26    | 48.3            | 45.6           | 41.9            | 38.9           | 35.6           | 30.4           | 25.3           | 20.8           | 17.3           | 15.4           | 13.8            | 12.2           | 11.2            |
| 27    | 49.6            | 47.0           | 43.2            | 40.1           | 36.7           | 31.5           | 26.3           | 21.7           | 18.1           | 16.2           | 14.6            | 12.9           | 11.8            |
| 28    | 51.0            | 48.3           | 44.5            | 41.3           | 37.9           | 32.6           | 27.3           | 22.7           | 18.9           | 16.9           | 15.3            | 13.6           | 12.5            |
| 29    | 52.3            | 49.6           | 45.7            | 42.6           | 39.1           | 33.7           | 28.3           | 23.6           | 19.8           | 17.7           | 16.0            | 14.3           | 13.1            |
| 30    | 53.7            | 50.9           | 47.0            | 43.8           | 40.3           | 34.8           | 29.3           | 24.5           | 20.6           | 18.5           | 16.8            | 15.0           | 13.8            |
| 40    | 66.8            | 63.7           | 59.3            | 55.8           | 51.8           | 45.6           | 39.3           | 33.7           | 29.1           | 26.5           | 24.4            | 22.2           | 20.7            |
| 50    | 79.5            | 76.2           | 71.4            | 67.5           | 63.2           | 56.3           | 49.3           | 42.9           | 37.7           | 34.8           | 32.4            | 29.7           | 28.0            |
| 60    | 92.0            | 88.4           | 83.3            | 79.1           | 74.4           | 67.0           | 59.3           | 52.3           | 46.5           | 43.2           | 40.5            | 37.5           | 35.5            |
| 70    | 104.2           | 100.4          | 95.0            | 90.5           | 85.5           | 77.6           | 69.3           | 61.7           | 55.3           | 51.7           | 48.8            | 45.4           | 43.3            |
| 80    | 116.3           | 112.3          | 106.6           | 101.9          | 96.6           | 88.1           | 79.3           | 71.1           | 64.3           | 60.4           | 57.2            | 53.5           | 51.2            |
| 90    | 128.3           | 124.1          | 118.1           | 113.1          | 107.6          | 98.6           | 89.3           | 80.6           | 73.3           | 69.1           | 65.6            | 61.8           | 59.2            |
| 100   | 140.2           | 135.8          | 129.6           | 124.3          | 118.5          | 109.1          | 99.3           | 90.1           | 82.4           | 77.9           | 74.2            | 70.1           | 67.3            |

Source: Catherine M. Thompson, *Table of percentage points of the  $\chi^2$  distribution*, Biometrika, Vol. 32 (1941), by permission of the author and publisher.

**95<sup>th</sup> Percentile Values  
for the *F* Distribution**  
( $v_1$  degrees of freedom in numerator)  
( $v_2$  degrees of freedom in denominator)

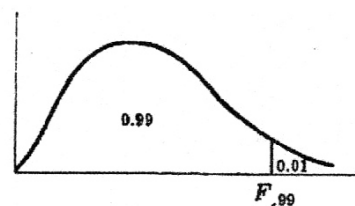


| $v_1 \backslash v_2$ | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 12   | 15   | 20   | 24   | 30   | 40   | 60   | 120  | $\infty$ |
|----------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|----------|
| 1                    | 161  | 200  | 216  | 225  | 230  | 234  | 237  | 239  | 241  | 242  | 244  | 246  | 248  | 249  | 250  | 251  | 252  | 253  | 254      |
| 2                    | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5     |
| 3                    | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.59 | 8.57 | 8.55 | 8.53     |
| 4                    | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.69 | 5.66 | 5.63     |
| 5                    | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.46 | 4.43 | 4.40 | 4.37     |
| 6                    | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 | 3.74 | 3.70 | 3.67     |
| 7                    | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 | 3.30 | 3.27 | 3.23     |
| 8                    | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 | 3.01 | 2.97 | 2.93     |
| 9                    | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 | 2.79 | 2.75 | 2.71     |
| 10                   | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 | 2.66 | 2.62 | 2.58 | 2.54     |
| 11                   | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 | 2.53 | 2.49 | 2.45 | 2.40     |
| 12                   | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.43 | 2.38 | 2.34 | 2.30     |
| 13                   | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 | 2.34 | 2.30 | 2.25 | 2.21     |
| 14                   | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 | 2.27 | 2.22 | 2.18 | 2.13     |
| 15                   | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.20 | 2.16 | 2.11 | 2.07     |
| 16                   | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 | 2.28 | 2.24 | 2.19 | 2.15 | 2.11 | 2.06 | 2.01     |
| 17                   | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 | 2.23 | 2.19 | 2.15 | 2.10 | 2.06 | 2.01 | 1.96     |
| 18                   | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 | 2.19 | 2.15 | 2.11 | 2.06 | 2.02 | 1.97 | 1.92     |
| 19                   | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 1.98 | 1.93 | 1.88     |
| 20                   | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.99 | 1.95 | 1.90 | 1.84     |
| 21                   | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.92 | 1.87 | 1.81     |
| 22                   | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.78     |
| 23                   | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.13 | 2.05 | 2.01 | 1.96 | 1.91 | 1.86 | 1.81 | 1.76     |
| 24                   | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.79 | 1.73     |
| 25                   | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.82 | 1.77 | 1.71     |
| 26                   | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.07 | 1.99 | 1.95 | 1.90 | 1.85 | 1.80 | 1.75 | 1.69     |
| 27                   | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.13 | 2.06 | 1.97 | 1.93 | 1.88 | 1.84 | 1.79 | 1.73 | 1.67     |
| 28                   | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.77 | 1.71 | 1.65     |
| 29                   | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.10 | 2.03 | 1.94 | 1.90 | 1.85 | 1.81 | 1.75 | 1.70 | 1.64     |
| 30                   | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.79 | 1.74 | 1.68 | 1.62     |
| 40                   | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.00 | 1.92 | 1.84 | 1.79 | 1.74 | 1.69 | 1.64 | 1.58 | 1.51     |
| 60                   | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.59 | 1.53 | 1.47 | 1.39     |
| 120                  | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.50 | 1.43 | 1.35 | 1.25     |
| $\infty$             | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.39 | 1.32 | 1.22 | 1.00     |

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table 5, page 178, by permission.



**99<sup>th</sup> Percentile Values  
for the *F* Distribution**  
( $v_1$  degrees of freedom in numerator)  
( $v_2$  degrees of freedom in denominator)



| $v_2 \backslash v_1$ | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 12   | 15   | 20   | 24   | 30   | 40   | 60   | 120  | $\infty$ |
|----------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|----------|
| 1                    | 4052 | 5000 | 5403 | 5625 | 5764 | 5859 | 5928 | 5981 | 6023 | 6056 | 6106 | 6157 | 6209 | 6235 | 6261 | 6287 | 6313 | 6339 | 6366     |
| 2                    | 98.5 | 99.0 | 99.2 | 99.2 | 99.3 | 99.3 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5     |
| 3                    | 34.1 | 30.8 | 29.5 | 28.7 | 28.2 | 27.9 | 27.7 | 27.5 | 27.3 | 27.2 | 27.1 | 26.9 | 26.7 | 26.6 | 26.5 | 26.4 | 26.3 | 26.2 | 26.1     |
| 4                    | 21.2 | 18.0 | 16.7 | 16.0 | 15.5 | 15.2 | 15.0 | 14.8 | 14.7 | 14.5 | 14.4 | 14.2 | 14.0 | 13.9 | 13.8 | 13.7 | 13.7 | 13.6 | 13.5     |
| 5                    | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.2 | 10.1 | 9.89 | 9.72 | 9.55 | 9.47 | 9.38 | 9.29 | 9.20 | 9.11 | 9.02     |
| 6                    | 13.7 | 10.9 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.72 | 7.56 | 7.40 | 7.31 | 7.23 | 7.14 | 7.06 | 6.97 | 6.88     |
| 7                    | 12.2 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.47 | 6.31 | 6.16 | 6.07 | 5.99 | 5.91 | 5.82 | 5.74 | 5.65     |
| 8                    | 11.3 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.67 | 5.52 | 5.36 | 5.28 | 5.20 | 5.12 | 5.03 | 4.95 | 4.86     |
| 9                    | 10.6 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.11 | 4.96 | 4.81 | 4.73 | 4.65 | 4.57 | 4.48 | 4.40 | 4.31     |
| 10                   | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.71 | 4.56 | 4.41 | 4.33 | 4.25 | 4.17 | 4.08 | 4.00 | 3.91     |
| 11                   | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.40 | 4.25 | 4.10 | 4.02 | 3.94 | 3.86 | 3.78 | 3.69 | 3.60     |
| 12                   | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.16 | 4.01 | 3.86 | 3.78 | 3.70 | 3.62 | 3.54 | 3.45 | 3.36     |
| 13                   | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.96 | 3.82 | 3.66 | 3.59 | 3.51 | 3.43 | 3.34 | 3.25 | 3.17     |
| 14                   | 8.86 | 6.51 | 5.56 | 5.04 | 4.70 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.80 | 3.66 | 3.51 | 3.43 | 3.35 | 3.27 | 3.18 | 3.09 | 3.00     |
| 15                   | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.67 | 3.52 | 3.37 | 3.29 | 3.21 | 3.13 | 3.05 | 2.96 | 2.87     |
| 16                   | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.55 | 3.41 | 3.26 | 3.18 | 3.10 | 3.02 | 2.93 | 2.84 | 2.75     |
| 17                   | 8.40 | 6.11 | 5.19 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.46 | 3.31 | 3.16 | 3.08 | 3.00 | 2.92 | 2.83 | 2.75 | 2.66     |
| 18                   | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.37 | 3.23 | 3.08 | 3.00 | 2.92 | 2.84 | 2.75 | 2.66 | 2.57     |
| 19                   | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.30 | 3.15 | 3.00 | 2.92 | 2.84 | 2.76 | 2.67 | 2.58 | 2.49     |
| 20                   | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.23 | 3.09 | 2.94 | 2.86 | 2.78 | 2.69 | 2.61 | 2.52 | 2.42     |
| 21                   | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.17 | 3.03 | 2.88 | 2.80 | 2.72 | 2.64 | 2.55 | 2.46 | 2.36     |
| 22                   | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.12 | 2.98 | 2.83 | 2.75 | 2.67 | 2.58 | 2.50 | 2.40 | 2.31     |
| 23                   | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 3.07 | 2.93 | 2.78 | 2.70 | 2.62 | 2.54 | 2.45 | 2.35 | 2.26     |
| 24                   | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.03 | 2.89 | 2.74 | 2.66 | 2.58 | 2.49 | 2.40 | 2.31 | 2.21     |
| 25                   | 7.77 | 5.57 | 4.68 | 4.18 | 3.86 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.99 | 2.85 | 2.70 | 2.62 | 2.54 | 2.45 | 2.36 | 2.27 | 2.17     |
| 26                   | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 2.96 | 2.82 | 2.66 | 2.58 | 2.50 | 2.42 | 2.33 | 2.23 | 2.13     |
| 27                   | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.93 | 2.78 | 2.63 | 2.55 | 2.47 | 2.38 | 2.29 | 2.20 | 2.10     |
| 28                   | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.90 | 2.75 | 2.60 | 2.52 | 2.44 | 2.35 | 2.26 | 2.17 | 2.06     |
| 29                   | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.87 | 2.73 | 2.57 | 2.49 | 2.41 | 2.33 | 2.23 | 2.14 | 2.03     |
| 30                   | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.84 | 2.70 | 2.55 | 2.47 | 2.39 | 2.30 | 2.21 | 2.11 | 2.01     |
| 40                   | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.66 | 2.52 | 2.37 | 2.29 | 2.20 | 2.11 | 2.02 | 1.92 | 1.80     |
| 60                   | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.50 | 2.35 | 2.20 | 2.12 | 2.03 | 1.94 | 1.84 | 1.73 | 1.60     |
| 120                  | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 | 2.47 | 2.34 | 2.19 | 2.03 | 1.95 | 1.86 | 1.76 | 1.66 | 1.53 | 1.38     |
| $\infty$             | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 | 2.32 | 2.18 | 2.04 | 1.88 | 1.79 | 1.70 | 1.59 | 1.47 | 1.32 | 1.00     |

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table 5, page 180, by permission.



Values of  $e^{-\lambda}$   
( $0 < \lambda < 1$ )

| $\lambda$ | 0      | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
|-----------|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0       | 1.0000 | .9900 | .9802 | .9704 | .9608 | .9512 | .9418 | .9324 | .9231 | .9139 |
| 0.1       | .9048  | .8958 | .8869 | .8781 | .8694 | .8607 | .8521 | .8427 | .8353 | .8270 |
| 0.2       | .8187  | .8106 | .8025 | .7945 | .7866 | .7788 | .7711 | .7634 | .7558 | .7483 |
| 0.3       | .7408  | .7334 | .7261 | .7189 | .7118 | .7047 | .6977 | .6907 | .6839 | .6771 |
| 0.4       | .6703  | .6636 | .6570 | .6505 | .6440 | .6376 | .6313 | .6250 | .6188 | .6126 |
| 0.5       | .6065  | .6005 | .5945 | .5886 | .5827 | .5770 | .5712 | .5655 | .5599 | .5543 |
| 0.6       | .5488  | .5434 | .5379 | .5326 | .5273 | .5220 | .5169 | .5117 | .5066 | .5016 |
| 0.7       | .4966  | .4916 | .4868 | .4819 | .4771 | .4724 | .4677 | .4630 | .4584 | .4538 |
| 0.8       | .4493  | .4449 | .4404 | .4360 | .4317 | .4274 | .4232 | .4190 | .4148 | .4107 |
| 0.9       | .4066  | .4025 | .3985 | .3946 | .3906 | .3867 | .3829 | .3791 | .3753 | .3716 |

( $\lambda = 1, 2, 3, \dots, 10$ )

| $\lambda$      | 1      | 2      | 3      | 4      | 5       | 6       | 7       | 8       | 9       | 10      |
|----------------|--------|--------|--------|--------|---------|---------|---------|---------|---------|---------|
| $e^{-\lambda}$ | .36788 | .13534 | .04979 | .01832 | .006738 | .002479 | .000912 | .000335 | .000123 | .000045 |

Note: To obtain values of  $e^{-\lambda}$  for other values of  $\lambda$ , use the laws of exponents.

Example:  $e^{-3.48} = (e^{-3.00})(e^{-0.48}) = (0.04979)(0.6188) = 0.03081$ .

Table 7. Critical Values of the Kolmogorov-Smirnov One Sample Test Statistics

This table gives the values of  $D_{n,\alpha}^+$  and  $D_{n,\alpha}^-$  for which  $\alpha \geq P\{D_n^+ > D_{n,\alpha}^+\}$  and  $\alpha \geq P\{D_n^- < D_{n,\alpha}^-\}$  for some selected values of  $n$  and  $\alpha$ .

| One-Sided Test : |      |      |      |      |      |            |            |            |            |            |            |
|------------------|------|------|------|------|------|------------|------------|------------|------------|------------|------------|
| $\alpha =$       | .10  | .05  | .025 | .01  | .005 | $\alpha =$ | .10        | .05        | .025       | .01        | .005       |
| Two-Sided Test : |      |      |      |      |      |            |            |            |            |            |            |
| $\alpha =$       | .20  | .10  | .05  | .02  | .01  | $\alpha =$ | .20        | .10        | .05        | .02        | .01        |
| n = 1            | .900 | .950 | .975 | .990 | .995 | n = 21     | .226       | .259       | .287       | .321       | .344       |
| 2                | .684 | .776 | .842 | .900 | .929 | 22         | .221       | .253       | .281       | .314       | .337       |
| 3                | .565 | .636 | .708 | .785 | .829 | 23         | .216       | .247       | .275       | .307       | .330       |
| 4                | .493 | .565 | .624 | .689 | .734 | 24         | .212       | .242       | .269       | .301       | .323       |
| 5                | .447 | .509 | .563 | .627 | .669 | 25         | .208       | .238       | .264       | .295       | .317       |
| 6                | .410 | .468 | .519 | .577 | .617 | 26         | .204       | .233       | .259       | .290       | .311       |
| 7                | .381 | .436 | .483 | .538 | .576 | 27         | .200       | .229       | .254       | .284       | .305       |
| 8                | .358 | .410 | .454 | .507 | .542 | 28         | .197       | .225       | .250       | .279       | .300       |
| 9                | .339 | .387 | .430 | .480 | .513 | 29         | .193       | .221       | .246       | .275       | .295       |
| 10               | .323 | .369 | .409 | .457 | .489 | 30         | .190       | .218       | .242       | .270       | .290       |
| 11               | .308 | .352 | .391 | .437 | .468 | 31         | .187       | .214       | .238       | .266       | .285       |
| 12               | .296 | .338 | .375 | .419 | .449 | 32         | .184       | .211       | .234       | .262       | .281       |
| 13               | .285 | .325 | .361 | .404 | .432 | 33         | .182       | .208       | .231       | .258       | .277       |
| 14               | .275 | .314 | .349 | .390 | .418 | 34         | .179       | .205       | .227       | .254       | .273       |
| 15               | .266 | .304 | .338 | .377 | .404 | 35         | .177       | .202       | .224       | .251       | .269       |
| 16               | .258 | .295 | .327 | .366 | .392 | 36         | .174       | .199       | .221       | .247       | .265       |
| 17               | .250 | .286 | .318 | .355 | .381 | 37         | .172       | .196       | .218       | .244       | .262       |
| 18               | .244 | .279 | .309 | .346 | .371 | 38         | .170       | .194       | .215       | .241       | .258       |
| 19               | .237 | .271 | .301 | .337 | .361 | 39         | .168       | .191       | .213       | .238       | .255       |
| 20               | .232 | .265 | .294 | .329 | .352 | 40         | .165       | .189       | .210       | .235       | .252       |
| Approximation    |      |      |      |      |      |            | 1.07       | 1.22       | 1.36       | 1.52       | 1.63       |
| For n > 40       |      |      |      |      |      |            | $\sqrt{n}$ | $\sqrt{n}$ | $\sqrt{n}$ | $\sqrt{n}$ | $\sqrt{n}$ |

Source. Adapted by permission from Table 1 of Leslie H. Miller. Table of Percentage points of Kolmogorov statistics, J. Am. Stat. Assoc. 51 (1956), 111-121.

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**SPACE FOR ROUGH WORK**

